Q8: 3.3-3.7

Graphs are always expected to show scale, with extrema and inflection points labelled on the graph. Graph paper or the provided grid must be used. On an exam, the instructions may just say, "Graph. Show all pertinent information such as intercepts, asymptotes local extema, inflection point", instead of detailed questions being asked on this quiz, but all this info is expected where applicable..

(1). For the function $f(x)=\frac{4}{x^2}-\frac{2}{x}+3 = \frac{4-2x+3x^2}{x^2}$

Find each of the following (if they exist). Show the work yielding your answers:

Domain: $(-\infty,0)\cup(0,\infty)$ denom $\neq 0 \Rightarrow x\neq 0$

x-intercept(s) : none $y=0 \Rightarrow 4-2x+3x^2=0 \Rightarrow x=\frac{2\pm\sqrt{4-48}}{6}$ no real soln.

y-intercept(s) : none $x=0$, not defined

Vertical Asymptotes, and limit approaching the vertical asymptote from each side. $x=0$ $\lim_{x\to 0^-}\frac{4-2x+3x^2}{x^2}=\infty$ $\frac{4}{0}$, $\lim_{x\to 0^+}\frac{4-2x+3x^2}{x^2}=\infty$

Horizontal Asymptote. $y=3$

$\lim_{x\to\infty} f(x)=3$

Points on the graph corresponding to local extrema (max or min?) local min at $(4, \frac{11}{4})$

$f(x)=4x^{-2}-2x^{-1}+3$

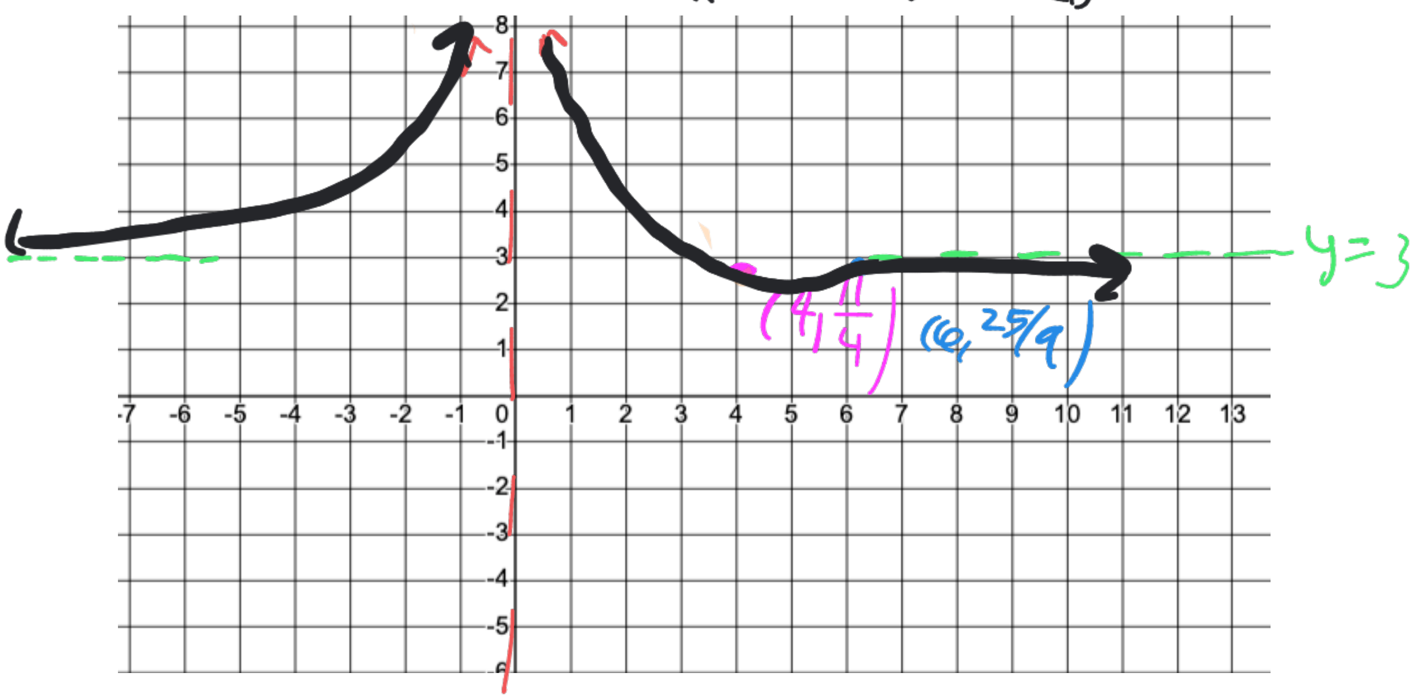
$f'(x)=-8x^{-3}+2x^{-2}=\frac{-8+2x}{x^3}$ $f'(x)=0 \Rightarrow x=4$ $f'(x)$ undef $\Rightarrow x=0$

$+ \quad - \quad +$ f' (0, 4)

$f''(x)=24x^{-4}-4x^{-3}$

Inflection Points $(6, 25/9)$

$f''(x)=24x^{-4}-4x^{-3}=\frac{24-4x}{x^4}$ $+ \quad -$ f'' 6 CU CD



Find the following. Be sure to show how you know it is really an absolute extreme. Make sure to specifically answer what is asked.

2) Where on the curve $f(x)=\dfrac{1}{1+x^2}$ does the tangent line have greatest slope?
Attach a computer graph which shows that your answer is reasonable.

Call the slope $m(x)$

$$m(x) = f'(x) = \frac{-2x}{(1+x^2)^2}$$

Maximize $m(x)$:

Find crit. #s

$$m'(x) = \frac{(1+x^2)^2(-2) + 2x \cdot 2(1+x^2)(2x)}{(1+x^2)^4}$$

$$m'(x) = \frac{(1+x^2)[-2(1+x^2) + 8x^2]}{(1+x^2)^4}$$

$$m'(x) = \frac{-2+6x^2}{(1+x^2)^3}$$

$$m'(x) = 0 \Rightarrow -2+6x^2 = 0$$

$$x^2 = 1/3$$

$$x = \pm 1/\sqrt{3}$$

$m(x)$: $+$ on $(-\infty, -\frac{1}{\sqrt{3}})$, $-$ on $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$, $+$ on $(\frac{1}{\sqrt{3}}, \infty)$

Max at $x=$